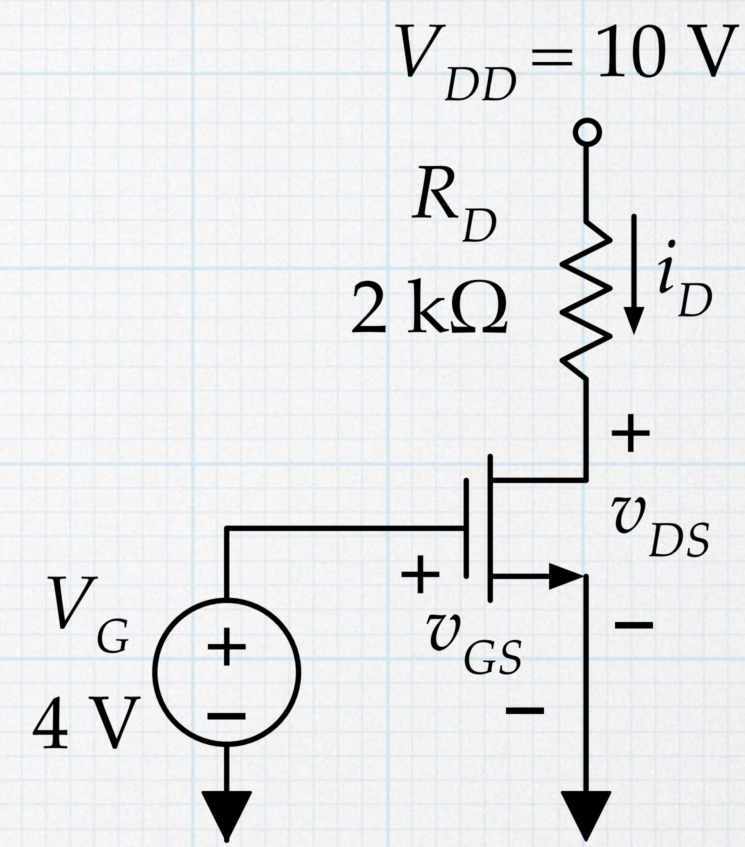
NMOS examples

For the circuit shown, use the the NMOS equations to find i_D and v_{DS}.



For the NMOS, V_T = 1.5 V and K = 0.5 mA/V^2.

$v_{GS} = V_G = 4$ V → the NMOS is on.

Assume that the transistor is in saturation.

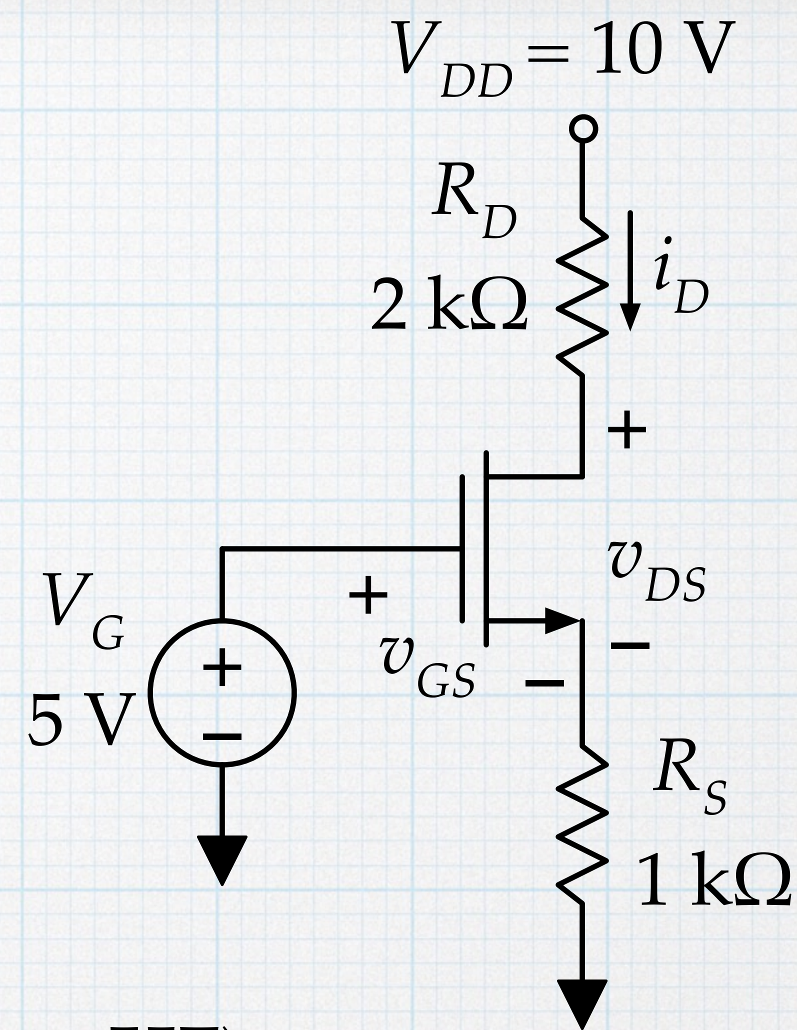
$$i_D = K\left(v_{GS} - V_T\right)^2 = \left(0.5\text{mA}/\text{V}^2\right)\left[4\text{V} - 1.5\text{V}\right]^2 = 3.125\text{mA}$$

$$v_{DS} = V_{DD} - i_D R_D = 10\text{V} - (3.125\text{mA})(2\text{k}\Omega) = 3.75\text{V}$$

$$v_{GS} - V_T = 4\text{ V} - 1.5\text{ V} = 2.5\text{ V}$$

$v_{DS} > v_{GS} - V_T$ → saturation confirmed. Q.E.D.

Example 2



For the circuit shown, use the the NMOS equations to find i_D and v_{DS}.

For the NMOS, $V_T = 1.0$ V and $K = 0.5$ mA/V^2.

Since $V_G > V_T$ → the NMOS is on.

Guess that the transistor is in saturation.

$$i_D = K\left(v_{GS} - V_T\right)^2$$

$$v_{GS} = V_G - i_S R_S \text{ (and } i_S = i_D\text{, as always for a FET)}$$

$$i_D = K\left(V_G - i_D R_S - V_T\right)^2$$

$$= K\left[R_S^2 i_D^2 - 2\left(V_G - V_T\right) R_S i_D + \left(V_G - V_T\right)^2\right]$$

Re-arranging:

$$i_D^2 - \left[\frac{1}{K R_S^2} + 2\left(\frac{V_G - V_T}{R_S}\right)\right] i_D + \left[\frac{V_G - V_T}{R_S}\right]^2 = 0$$

$$i_D^2 - \left[\frac{1}{KR_S^2} + 2\left(\frac{V_G - V_T}{R_S}\right)\right] i_D + \left[\frac{V_G - V_T}{R_S}\right]^2 = 0$$

Plug in the numbers:

$$i_D^2 - [10\text{ mA}]\, i_D + \left[9\text{ mA}^2\right] = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic equation:

or, if $a = 1$

$$x = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

i_D = 9 mA or i_D = 1 mA.

Which is right? Check the v_{GS} for both.

If i_D = 9 mA, $v_{GS} = V_G - i_D R_S$ = 4 V– 9 V = –5 V. No way! The NMOS would not be on in that case. This root cannot be correct.

If i_D = 1 mA, $v_{GS} = V_G - i_D R_S$ = 4 V– 1 V = 3 V. OK, this is consistent.

Finally, $v_{GS} - V_T = V_G - i_D R_S - V_T$ = 2 V, and $v_{DS} = V_{DD} - i_D R_D - i_D R_S$ = 7 V.

$v_{DS} > v_{GS} - V_T \rightarrow$ saturation confirmed.

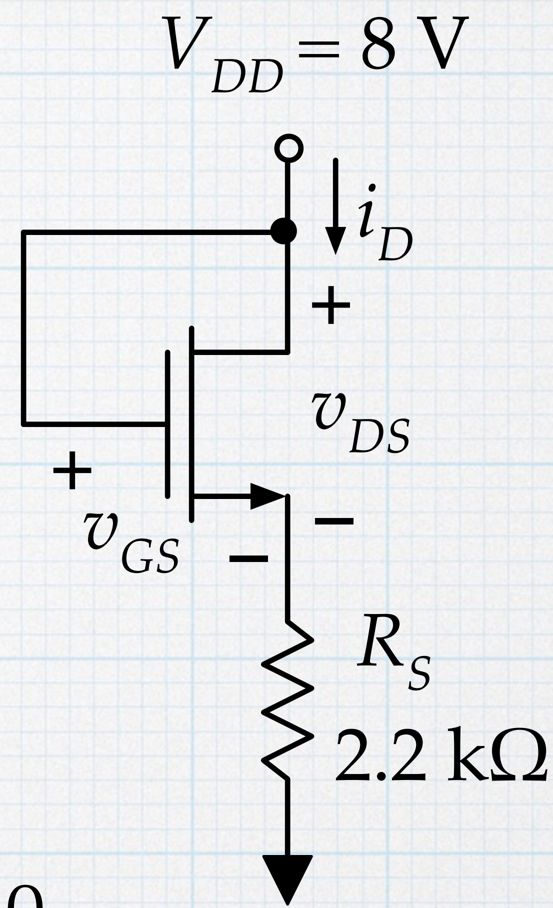
Example 3

For the circuit shown, use the the NMOS equations to find i_D and v_{DS}.

For the NMOS, V_T = 1.5 V and K = 0.25 mA/V^2.

First note that $v_{GS} = v_{DS}$, so the NMOS *must* be in saturation. ($v_{DS} > v_{GS} - V_T$) No guess needed.

Since $V_{DD} > V_T$ → the NMOS is on.



And $i_S = i_D$ (always for a FET) and don't forget that $i_G = 0$.

$$i_D = K\left(v_{GS} - V_T\right)^2 \qquad v_{GS} = V_{DD} - i_S R_S$$

$$i_D = K\left(V_{DD} - i_D R_S - V_T\right)^2$$

$$= K\left[R_S^2 i_D^2 - 2\left(V_{DD} - V_T\right) R_S i_D + \left(V_{DD} - V_T\right)^2\right]$$

(Same basic form as Example 2.)

$$i_D^2 - \left[\frac{1}{KR_S^2} + 2\left(\frac{V_{DD} - V_T}{R_S}\right)\right] i_D + \left[\frac{V_{DD} - V_T}{R_S}\right]^2 = 0$$

$$i_D^2 - \left[\frac{1}{KR_S^2} + 2\left(\frac{V_{DD} - V_T}{R_S}\right)\right] i_D + \left[\frac{V_{DD} - V_T}{R_S}\right]^2 = 0$$

Plug in the numbers:

$$i_D^2 - [6.74 \text{ mA}]\, i_D + \left[8.73 \text{ mA}^2\right] = 0$$

Use the quadratic equation:

i_D = 4.99 mA or i_D = 1.75 mA.

Which is right? Check the v_{GS} for both.

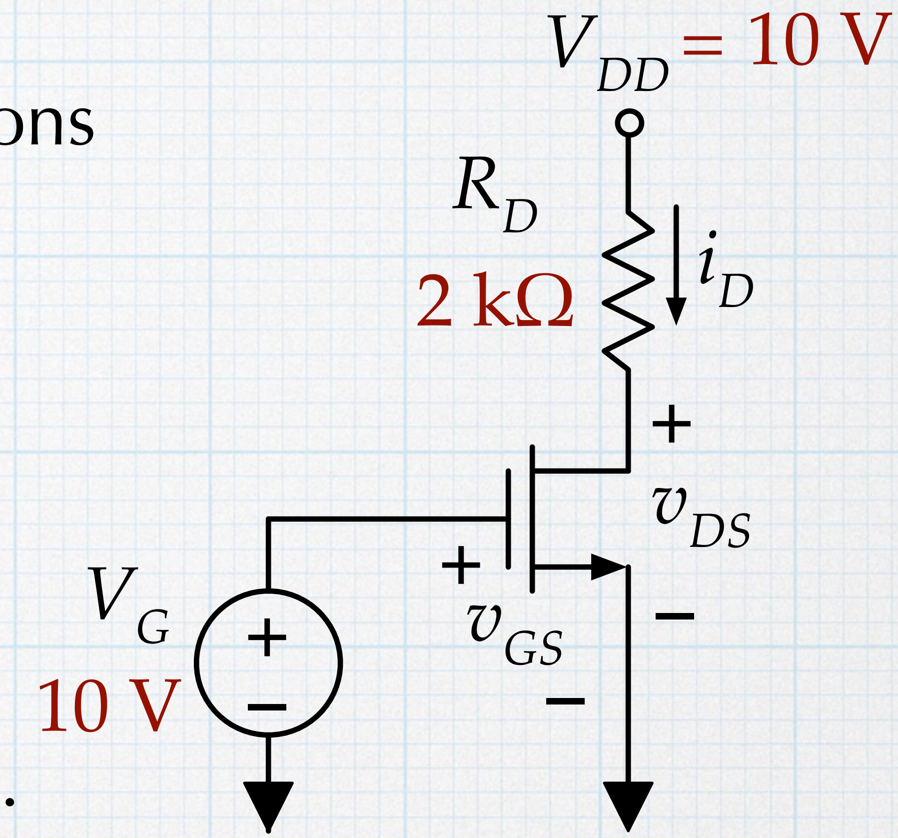
If i_D = 4.99 mA, $v_{GS} = V_{DD} - i_D R_S$ = 8 V– 10.98 V = –2.98 V. Nope – the NMOS would not be on in that case. This root is bogus.

If i_D = 1.75 mA, $v_{GS} = V_{DD} - i_D R_S$ = 8 V– 3.85 V = 4.15 V. OK, this works.

Finally, $v_{DS} = v_{GS}$ = 4.15 V.

Example 4

For the circuit shown, use the the NMOS equations to find i_D and v_{DS}.



For the NMOS, V_T = 1.5 V and K = 0.5 mA/V^2.

$v_{GS} = V_G$ = 10 V → the NMOS is on.

This looks like a lot like the first example. So start by assuming that the NMOS is in saturation.

$$i_D = K\left(v_{GS} - V_T\right)^2 = \left(0.5\text{mA}/\text{V}^2\right)\left[10\text{V} - 1.5\text{V}\right]^2 = 36.125\text{mA}$$

$$v_{DS} = V_{DD} - i_D R_D = 10\text{V} - (36.125\text{mA})\,(2\text{k}\Omega) = -62.25\text{V}$$

Red Alert! There is a serious problem here. Apparently the NMOS is **not** in saturation. So try the ohmic equation.

$$i_D = K\left[2\left(v_{GS} - V_T\right)v_{DS} - v_{DS}^2\right]$$

Unfortunately, we don't know either i_D or v_{DS}. So we need a second equation.

$$i_D = K\left[2\left(v_{GS} - V_T\right)v_{DS} - v_{DS}^2\right]$$

Use Ohm's law on the drain resistor to get a second equation:

$$i_{RD} = \frac{V_{DD} - v_{DS}}{R_D} = i_D$$

We can use these to solve for either i_D or v_{DS}. Setting the two equal and solving for v_{DS} is probably slightly easier.

$$\frac{V_{DD} - v_{DS}}{R_D} = K\left[2\left(v_{GS} - V_T\right)v_{DS} - v_{DS}^2\right]$$

Re-arrange: $$v_{DS}^2 - \left[2\left(v_{GS} - V_T\right) + \frac{1}{KR_D}\right]v_{DS} + \frac{V_{DD}}{KR_D} = 0$$

Plug in numbers: $v_{DS}^2 - [18\text{ V}]\,v_{DS} + 10\text{ V}^2 = 0$

Solve: v_{DS} = 0.574 V or v_{DS} = 17.43 V.

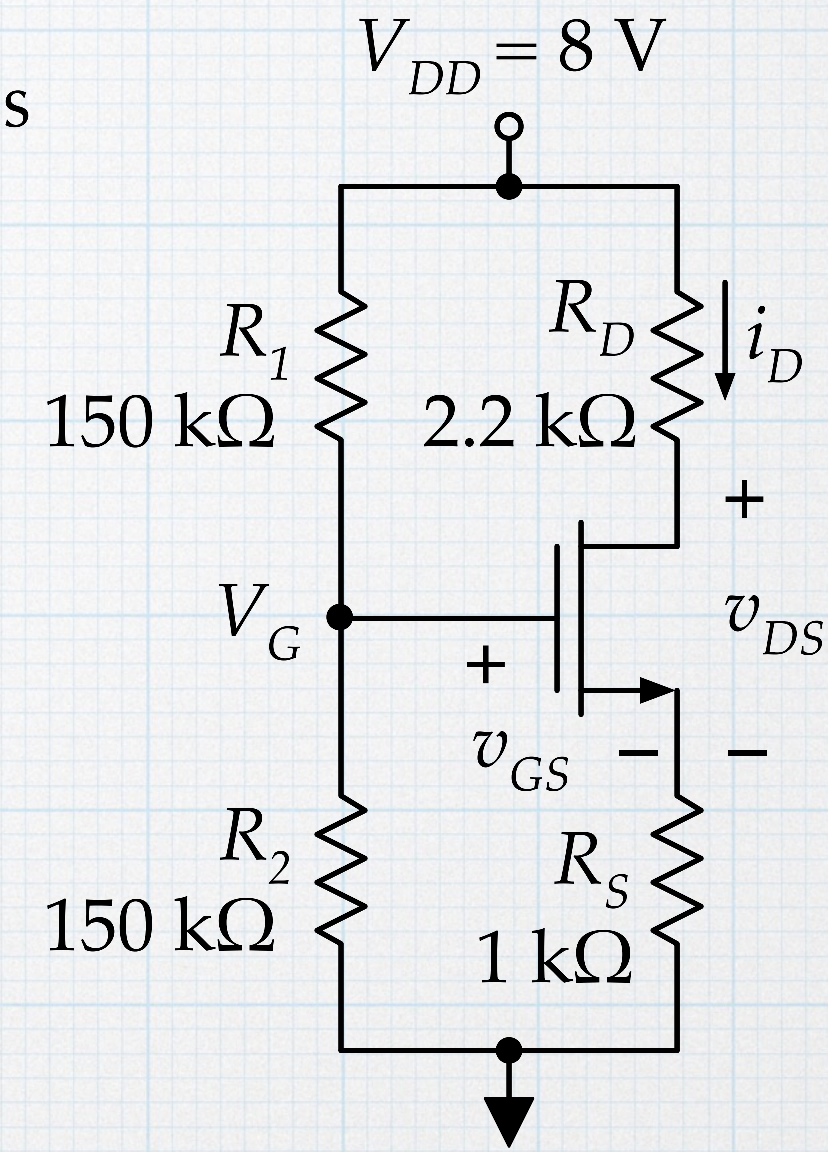
It should be obvious that the larger value is way too big – it's bigger than V_{DD}. Also, since the NMOS is in ohmic, we expect v_{DS} to be small. So we choose the smaller value as correct.

Lastly: $$i_D = \frac{V_{DD} - v_{DS}}{R_D} = \frac{10\text{ V} - 0.573\text{ V}}{2\text{k}\Omega} = 4.71\text{ mA}$$

Example 5

For the circuit shown, use the the NMOS equations to find i_D and v_{DS}.

For the NMOS, V_T = 1 V and K = 0.25 mA/V^2.



First note that since $i_G = 0$, R_1 and R_2 form a simple voltage divider, and

$$V_G = \frac{R_2}{R_2 + R_1} V_{DD} = 4 \text{ V}$$

Since $V_G > V_T$, the NMOS should be on. Guess that it is in saturation.

$$v_{GS} = V_G - v_{RS} = V_G - i_D R_S$$

$$i_D = K\left(v_{GS} - V_T\right)^2 = K\left(V_G - i_D R_S - V_T\right)^2$$

This is exactly the same as example 2.

$$i_D^2 - \left[\frac{1}{KR_S^2} + 2\left(\frac{V_G - V_T}{R_S}\right)\right] i_D + \left[\frac{V_G - V_T}{R_S}\right]^2 = 0$$

$$i_D^2 - \left[\frac{1}{KR_S^2} + 2\left(\frac{V_G - V_T}{R_S}\right)\right] i_D + \left[\frac{V_G - V_T}{R_S}\right]^2 = 0$$

Plug in the numbers:

$$i_D^2 - [10\ \text{mA}]\, i_D + 9\ \text{mA}^2 = 0$$

Use the quadratic equation:

i_D = 9 mA or i_D = 1 mA.

Which is right? Check the v_{GS} for both.

If $i_D = 9$ mA, $v_{GS} = V_G - i_D R_S$ = 4 V – 8.24 V = –5 V, and
if $i_D = 1$ mA, $v_{GS} = V_G - i_D R_S$ = 4 V– 1 V = 3 V.

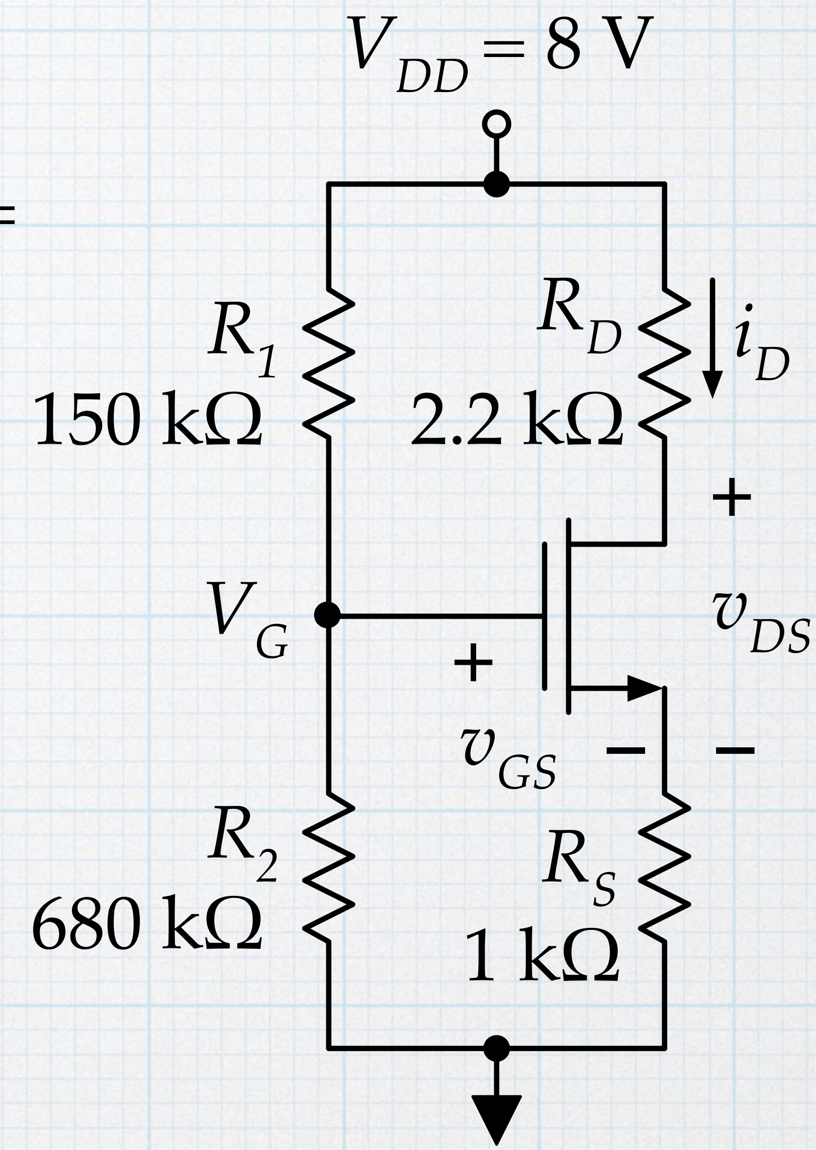
Clearly, $i_D = 1$ mA is the only answer that makes sense.

Finally, $v_{GS} - V_T$ = 2 V, and $v_{DS} = V_{DD} - i_D R_D - i_D R_S$ = 4.8 V.

$v_{DS} > v_{GS} - V_T \rightarrow$ saturation confirmed.

Example 6

Same as example 5, but values for R_2 is increased to 680 kΩ. It is the same NMOS: V_T = 1 V and K = 0.25 mA/V^2.



Following the same procedure as Example 5, we obtain $V_G = 6.55$ V. Guessing saturation and performing the same calculation to find i_D,

$$i_D = 2.44 \text{ mA or } i_D = 12.7 \text{ mA.}$$

Again, the larger of these is clearly too big to make any sense. Checking the smaller value for consistency with saturation:

$$v_{GS} - V_T = V_G - i_D R_S - V_T = 3.11 \text{ V, and}$$

$$v_{DS} = V_{DD} - i_D R_D - i_D R_S = 0.19 \text{ V.}$$

Oops!! $v_{DS} < v_{GS} - V_T$ → This is not in saturation!

So start over, assuming ohmic operation:

$$i_D = K\left[2\left(v_{GS} - V_T\right)v_{DS} - v_{DS}^2\right]$$

$$v_{GS} - V_T = V_G - i_D R_S - V_T$$

$$v_{DS} = V_{DD} - i_D R_D - i_D R_S$$

This is gonna be messy…

$$\frac{i_D}{K} = 2\left[V_G - i_D R_S - V_T\right]\left[V_{DD} - i_D\left(R_D + R_S\right)\right] - \left[V_{DD} - i_D\left(R_D + R_S\right)\right]^2$$

After a whole lotta algebra…

$$i_D^2 - \left[\frac{1}{K\left(R_S^2 - R_D^2\right)} + \frac{2R_S V_{DD}}{\left(R_S^2 - R_D^2\right)} + \frac{2\left(V_G - V_t\right)}{\left(R_S - R_D\right)} - \frac{2V_{DD}}{\left(R_S - R_D\right)}\right] i_D$$

(Ouch! That one hurt…)

$$+\left[\frac{2\left(V_G - V_T\right)V_{DD} - V_{DD}^2}{\left(R_S^2 - R_D^2\right)}\right] = 0$$

Plug in the numbers: $i_D^2 + [1.12 \text{ mA}]\, i_D - 6.45 \text{ mA}^2 = 0$

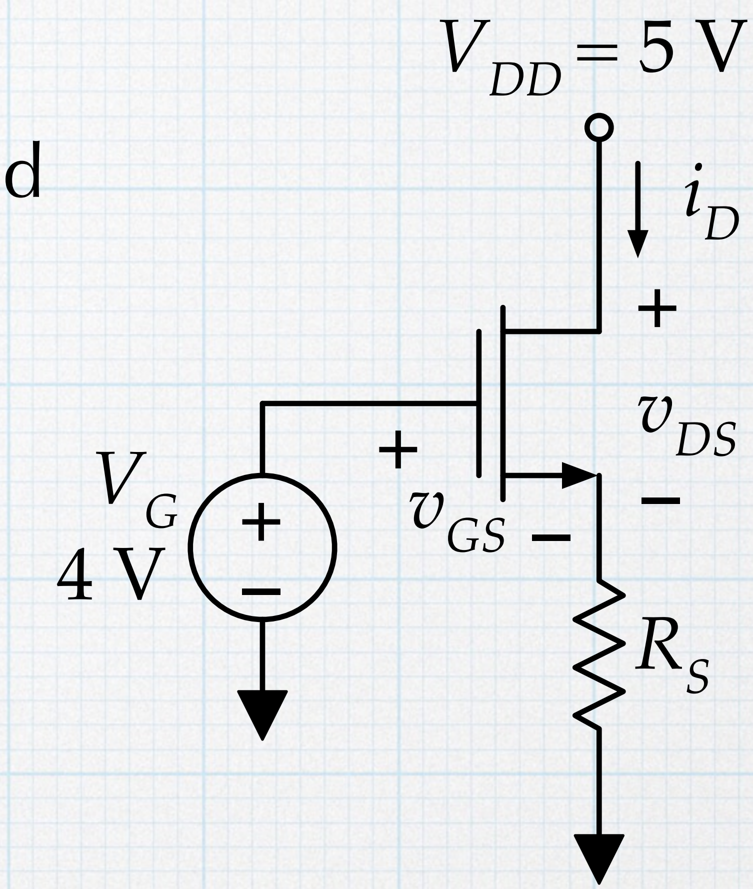
and the two roots are: $i_D = 2.04$ mA and $i_D = \cancel{-3.16 \text{ mA}}$.

Definitely ohmic.

$$v_{DS} = 8 \text{ V} - (2.04 \text{ mA})(2.2 \text{ k}\Omega + 1 \text{ k}\Omega) = 1.47 \text{ V}$$

Example 7

Design the circuit at right (by choosing K for the NMOS and the value of R_S) so that $i_D = 1$ mA and $v_{DS} = 2.5$ V. The NMOS has $V_T = 1$ V.



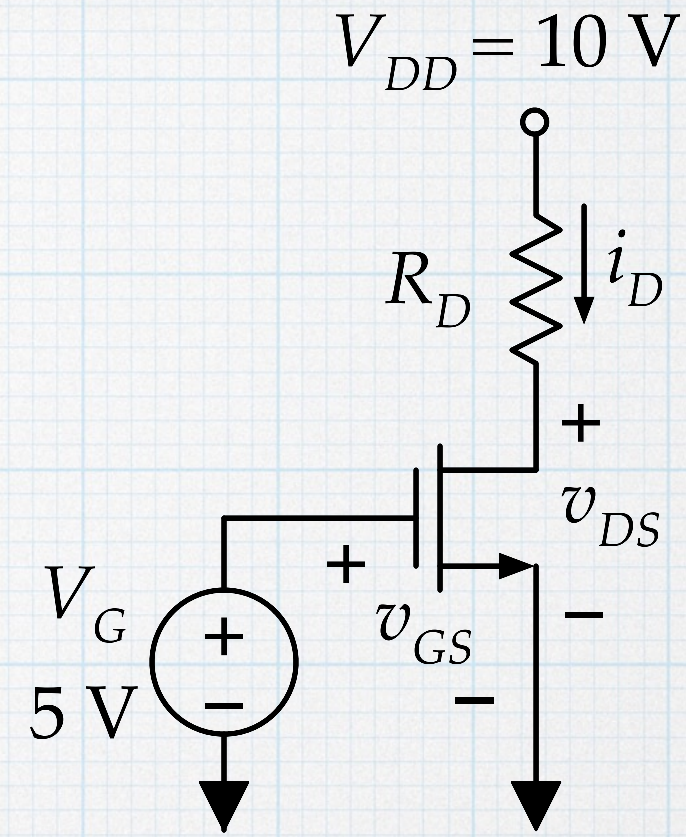
By writing a loop equation around the drain-source loop, we see that $v_{RS} = V_{DD} - v_{DS} = 2.5$ V. And so $R_S = 2.5\text{ V} / 1\text{ mA} = 2.5\text{ k}\Omega$.

Now writing a loop equation around the gate-source loop, we see that $v_{GS} = V_G - v_{RS} = 1.5$ V. This value of v_{GS} means that the NMOS must be operating in saturation.

Then, since in saturation $i_D = K(v_{GS} - V_T)^2$,

$$K = \frac{i_D}{(v_{GS} - V_T)^2} = \frac{1\text{ mA}}{(1.5\text{ V} - 1\text{ V})^2} = 4\frac{\text{mA}}{\text{V}^2}$$

Example 8



Design the circuit at right (by choosing K for the NMOS and the value of R_D) so that $i_D = 10$ mA and $v_{DS} = 0.2$ V. The NMOS has $V_T = 1$ V. How much power is being dissipated in the resistor and the NMOS?

If $v_{DS} = 0.2$ V, then $v_{RD} = 9.8$ V. For a current of 10 mA, $R_D = v_{RD} \,/\, i_D = 9.8\text{ V}/10\text{mA} = 0.98\text{ k}\Omega$.

With $v_{GS} = 5$ V and $v_{DS} = 0.2$ V, the NMOS must be working in the ohmic region. For ohmic operation:

$$i_D = K\left[2\left(v_{GS} - V_T\right)v_{DS} - v_{DS}^2\right]$$

$$K = \frac{i_D}{\left[2\left(v_{GS} - V_T\right)v_{DS} - v_{DS}^2\right]} = \frac{10\text{ mA}}{2\left(5\text{ V} - 1\text{ V}\right)\left(0.2\text{ V}\right) - \left(0.2\text{ V}\right)^2} = 6.41\,\frac{\text{mA}}{\text{V}^2}$$

$$P_{RD} = \left(9.8\text{ V}\right)\left(10\text{ mA}\right) = 98\text{ mW} \qquad P_{NMOS} = \left(0.2\text{ V}\right)\left(10\text{ mA}\right) = 2\text{ mW}$$